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Perturbative Odderon in the Dipole Model*

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Abstract

We show that, in the framework of Mueller's dipole model, the perturbative QCD odderon is described by the dipole model equivalent of the BFKL equation with a C-odd initial condition. The eigenfunctions and eigenvalues of the odderon solution are the same as for the dipole BFKL equation and are given by the functions $E^{n,\nu}$ and $\chi(n,\nu)$ correspondingly, where the C-odd initial condition allows only for odd values of n. The leading high-energy odderon intercept is given by $\alpha_{odd} - 1 = \frac{2\,\alpha_s\,N_c}{\pi}\,\chi(n=1,\nu=0) = 0$ in agreement with the solution found by Bartels, Lipatov and Vacca. We proceed by writing down an evolution equation for the odderon including the effects of parton saturation. We argue that saturation makes the odderon solution a decreasing function of energy.

*We dedicate this work to the memory of Jan Kwieciński.

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1 Introduction

The dominant contributions to the total cross sections of hadronic reactions are related to the Pomeron and to the Odderon exchanges. While the Pomeron exchange having the quantum numbers of the vacuum represents a dominant contribution to the sum of total cross sections for a given hadronic process and its crossing counterpart which is even under crossing, the Odderon exchange dominates the difference of these two cross sections, which is odd under the crossing symmetry. Within Regge theory Pomeron and Odderon are therefore natural partners. The importance of Odderon exchange for the phenomenology of hadronic reactions was noted long ago [1] and was the subject of many investigations. A review of these early studies can be found in, e.g., [2].

With the advent of quantum chromodynamics (QCD) theoretical status of the perturbative Odderon followed the development of the analogous description of its Pomeron partner. Within QCD in the leading logarithmic approximation (LLA), the Odderon appears as the color singlet exchange of three gluons, which interactions are of a form very similar to the interactions of the two gluons forming the Pomeron. It is therefore not surprising that soon after the derivation of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [3] for the Pomeron there was derived an analogous Bartels-Kwiecinski-Praszalowicz [4] equation for the Odderon.

The solution of the BFKL equation by Lipatov in the seminal Ref. [5] stimulated similar studies of the BKP equation which, because of its three-body nature, turned out to be much more difficult to solve. These attempts led nevertheless to a discovery of a deep connection between the BFKL equation, the BKP equation and their generalizations on one side and the two dimensional conformal models of spin chains [6] and their exact solubility [7, 8] on another side. Finally they also led to discovery of several solutions of the BKP equation [9], [10], [11] with different intercepts of the corresponding Odderon Regge trajectories determining the asymptotics of the cross sections. In particular, the solution found by Bartels, Lipatov and Vacca (BLV) in Ref. [11] describes the Odderon with the largest intercept found to date, which is equal to 1*. Thus the present results suggest that the perturbative Odderon intercept is smaller than the one for the BFKL Pomeron.

From an experimental point of view the Odderon remains a mystery. Recent data on the differential elastic pp scattering show that one needs the Odderon to describe the cross sections in the dip region [13]. On the other hand, the perturbative QCD prediction in the Born approximation for the cross section of diffractive photoproduction of η_c-meson is rather small [14]. The inclusion of evolution based on the BKP equation increases the cross section by one order of magnitude [15]. Because of that suggestions were raised to look for Odderon effects in the studies of the charge asymmetries in the two pion diffractive production [16] and η_c electroproduction in the triple Regge region [17]. A comprehensive survey of searches for Odderon effects as well as its present day theoretical status is given in the recent review [18].

The original derivation of the BFKL equation was based on the LLA resummation of conventional Feynman diagrams in the Regge kinematics typical for high energy and diffractive processes. Another approach to the description of high energy processes was proposed by Mueller, who constructed the dipole model [19] based on the light cone perturbation theory. In

*Here we restrict ourselves to the Odderon described by the BKP equation for three gluons in the t-channel. In Ref. [12] it was argued that exchanges of many t-channel gluons generalizing the BKP equation may also lead to the Odderon intercept asymptotically approaching 1 from below.

the dipole model, the Pomeron, or in other words the BFKL evolution emerges as an evolution of the wave function of a projectile particle as viewed in the reference frame of a target particle. In Ref. [20] Navelet and Wallon have shown the equivalence of the color dipole model and the BFKL Pomeron. Since the dipole model is actually a very popular and efficient tool for the description of high energy and diffractive processes and a starting point for inclusion of saturation effects [21, 22] it appears natural to ask whether the Odderon can also be incorporated in the dipole model similar to how the BKP equation was derived within the same approach that had earlier led to the BFKL equation. Finding the answer to this question is the main motivation for the present work.

The paper is structured as follows. In Sect. 2 we demonstrate that the Odderon evolution is described in the dipole model by the same evolution equation (7) as was used in [19] to describe the BFKL evolution. The only difference is that to project out the Odderon evolution from this equation one needs to impose C-odd initial conditions (8). The solution of Eq. (7) is given by Eq. (14) using the eigenfunctions (9) and the eigenvalues (10) for odd n. The leading high energy Odderon intercept is equal to 1 as shown in Eq. (19). In Sect. 3 we show that our solution (18) is equivalent to the BLV Odderon solution [11]. Thus, for a given three-gluon initial condition, the dipole BFKL equation is equivalent to the BKP equation for three t-channel gluons. In Sect. 4 we write down the equation (26) describing the Odderon evolution including the saturation effects. Saturation is likely to make the Odderon solution a decreasing function of energy. We conclude in Sect. 5 by discussing what we have and have not proved.

2 Evolution Equation For Perturbative QCD Odderon in the Dipole Model

In this Section we show that in Mueller's dipole model the odderon evolution equation is given by the dipole model equivalent of the BFKL equation taken with a C-odd initial condition. Solutions of this equation for the odderon exchange amplitude are given by functions $E^{n,\nu}$ with odd n. Corresponding eigenvalues are given by the BFKL equation eigenvalues $\chi(n,\nu)$ with odd n, such that the leading high energy intercept is given by $\alpha_{odd} - 1 = \frac{2\,\alpha_s\,N_c}{\pi}\,\chi(n=1,\nu=0) = 0$.

2.1 One Step of Small-x Evolution

Let us consider diffractive scattering of a $q\bar{q}$ dipole on some target, which could be another dipole for the case of onium–onium scattering or a proton for the case of Deep Inelastic Scattering (DIS). For the scattering amplitude of a $q\bar{q}$ dipole on a target the operation of charge conjugation corresponds to interchanging the quark and the anti-quark lines. Consider a dipole consisting of a quark at transverse coordinate $\underline{x}_0$ and an anti-quark at transverse coordinate $\underline{x}_1$ carrying the light cone momentum fractions z and $1-z$ of the total light cone momentum of the $q\bar{q}$ system. Operation of charge conjugation corresponds to replacing

$$C: \qquad \underline{x}_0 \leftrightarrow \underline{x}_1, \qquad z \leftrightarrow 1-z. \tag{1}$$

Odderon exchange, by definition, corresponds to diffractive amplitudes which are anti-symmetric under the operation shown in Eq. (1):

$$\mathcal{O}(\underline{x}_0, \underline{x}_1; z, 1-z) \;=\; -\,\mathcal{O}(\underline{x}_1, \underline{x}_0; 1-z, z), \tag{2}$$

where $\mathcal{O}(\underline{x}_0, \underline{x}_1; z, 1-z)$ is the forward amplitude of a dipole 01 on some target. Below we will use Eq. (2) as a definition of the odderon–mediated amplitude.

Our goal now is to start with some C-odd initial condition for onium–target scattering process and try to include one step of the dipole evolution [19, 23] in it. The simplest initial condition is given by the color-singlet three-gluon exchange diagram shown in Fig. 1 corresponding to the lowest order odderon exchange. Here, for simplicity, we take the target to be a single quark line at the bottom of Fig. 1 located at transverse coordinate $\underline{x} = \underline{0}$. In Fig. 1 the disconnected t-channel gluon lines imply summation over all possible connections of these lines to both the quark and the anti-quark in the onium state at the top of the figure, as shown in Fig. 5. When convoluted with virtual photon's wave function on the left hand side and with a meson's wave function on the right hand side, the diagram in Fig. 1 would contribute to production of pions, η_c, etc., in DIS (see e.g. [11, 15]).

The three-gluon exchange diagram in Fig. 1 gives the imaginary part of the forward odderon-mediated amplitude ($x_i = |\underline{x}_i|$)

$$\mathcal{O}_0(\underline{x}_0, \underline{x}_1) \, = \, c_0 \, \alpha_s^3 \, \ln^3 \frac{x_0}{x_1} \tag{3}$$

with

$$c_0 = \frac{(N_c^2 - 4)\,(N_c^2 - 1)}{4\, N_c^3}. \tag{4}$$

The color structure of the color-singlet three gluon exchange is given by d^{abc} for the odderon amplitude. The calculation leading to Eq. (3) can be performed in covariant gauge, as well as in the $A^+ = 0$ light cone gauge which we will be using throughout the paper. (The onium is moving in the light cone + direction.) One can easily see that the amplitude in Eq. (3) satisfies the condition of Eq. (2) and is, therefore, C-odd. One can also verify that the integral of the amplitude (3) over the impact parameter $\underline{b} = (\underline{x}_0 + \underline{x}_1)/2$ is zero, in agreement with the fact that the odderon exchange is zero at momentum transfer $t = 0$.

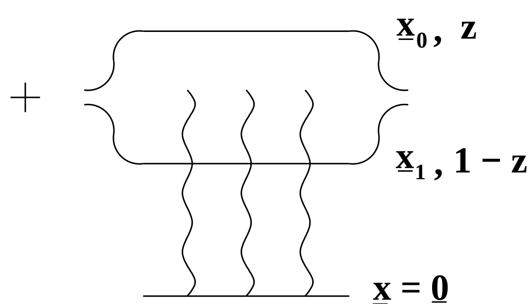

Figure 1: Lowest order odderon exchange diagram which will be used as initial condition for small-x evolution. The gluon lines connect to both the quark and the anti-quark in the onium above.

One step of the dipole evolution [19] corresponds to emitting a single s-channel gluon in the onium wave function. In the large-N_c limit in which the dipole model [19] is constructed, the gluon line is represented by a double quark line, as shown in Fig. 2. The diagram in Fig. 2 represents a real part of the dipole kernel, where the emitted gluon is present at the time of

scattering on the target. The s-channel gluon can be emitted by either quark or anti-quark to the left and to the right of the interaction, which is implied by the disconnected gluon (double) line. The original dipole 01 is split into two (color-singlet) dipoles 02 and 21. Three exchanged gluons can only couple all together to either one of the dipoles 02 and 21. Fig. 2 shows the interaction with the dipole 21. The only other possible interaction diagram involves one of the gluons coupling to one of the dipoles with the other two gluons coupling to another dipole. That diagram is zero since a color-neutral dipole can not interact with the target by a single gluon exchange.

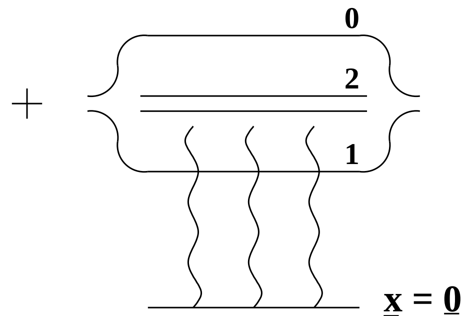

Figure 2: One step of dipole evolution with the three-gluon exchange initial conditions.

Employing the dipole evolution kernel from [19] we obtain the following expression for the first step of dipole evolution with the initial condition (3) ($\underline{x}_{ij} = \underline{x}_i - \underline{x}_j$)

$$\frac{\alpha_s\, N_c}{2\,\pi^2} \int_{z_{init}}^{\min\{z,1-z\}} \frac{dz_2}{z_2} \int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}\, \left[\mathcal{O}_0(\underline{x}_0, \underline{x}_2) + \mathcal{O}_0(\underline{x}_2, \underline{x}_1) - \mathcal{O}_0(\underline{x}_0, \underline{x}_1)\right] \tag{5}$$

where the first and the second terms correspond to real emissions with the second term pictured in Fig. 2, while the last term in Eq. (5) corresponds to the virtual correction [19]. In Eq. (5) z_2 is the fraction of onium's longitudinal ("plus") momentum carried by the gluon 2, which, in the leading logarithmic approximation, is integrated from some initial value z_{init} up to the smaller one of the light cone momentum fractions carried by the original quark and anti-quark [19]. Substituting $\mathcal{O}_0$ from Eq. (3) into Eq. (5) yields

$$3\,\frac{\alpha_s\, N_c}{2\,\pi^2}\, c_0\, \alpha_s^3\, \ln\frac{x_0}{x_1}\, \ln\left(\frac{\min\{z, 1-z\}}{z_{init}}\right) \int d^2x_2\, \frac{x_{01}^2}{x_{02}^2\, x_{12}^2}\, \ln\frac{x_0}{x_2}\, \ln\frac{x_1}{x_2}. \tag{6}$$

The amplitude in Eq. (6) is non-zero. It changes sign under the transformation of Eq. (1) and is thus C-odd. We conclude that C-odd initial conditions leave a dipole amplitude C-odd even after one step of small-x dipole evolution. The resulting amplitude is non-zero, which means that the projection of the dipole evolution on the C-odd channel is non-trivial.

2.2 Evolution Equation

It is now straightforward to write down an evolution equation for the odderon in the dipole model. We have shown above that in the dipole model a single step of the small-x evolution for

the odderon is the same as for the BFKL pomeron [19]. Subsequent steps of the dipole evolution would generate more color dipoles in the onium wave function. Similar to one step of evolution shown in Fig. 2, there is only one way of connecting the three-gluon initial conditions to the fully evolved dipole wave function having many dipoles in it: only the diagrams with all three gluons interacting with the same dipole survive, since a dipole can not interact via a single gluon exchange. Therefore, the picture of the odderon evolution is simple. The onium wave function develops the usual dipole cascade described by the leading logarithmic dipole evolution [19]. In the linear evolution approximation, one of the dipoles generated by the evolution interacts with the target. In the case of the pomeron exchange, the lowest order interaction is given by a two-gluon exchange [19]. For the case of the odderon considered here, the interaction should be C-odd and at the lowest order is given by the three-gluon exchange amplitude (3).

For the odderon evolution equation we thus write [19]

$$\frac{\partial}{\partial Y}\mathcal{O}(\underline{x}_0, \underline{x}_1, Y) \, = \, \frac{\alpha_s \, N_c}{2\,\pi^2} \int d^2 x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} \, [\mathcal{O}(\underline{x}_0, \underline{x}_2, Y) + \mathcal{O}(\underline{x}_2, \underline{x}_1, Y) - \mathcal{O}(\underline{x}_0, \underline{x}_1, Y)] \tag{7}$$

where we changed the arguments of the dipole amplitude $\mathcal{O}(\underline{x}_i, \underline{x}_j; z_i, z_j)$ to $\mathcal{O}(\underline{x}_i, \underline{x}_j, Y)$ such that $Y = \ln(\min\{z_i, z_j\}/z_{init})$ is the rapidity variable. In the leading logarithmic approximation for gluon evolution the dependence of the dipole amplitude on the light cone momentum fractions of the quark z_i and the antiquark z_j comes in only through the quantity $\min\{z_i, z_j\}$ [19], which allowed us to reduce the number of arguments in $\mathcal{O}$.

The initial condition for the Eq. (7) is given by

$$\mathcal{O}(\underline{x}_0, \underline{x}_1, Y = 0) \, = \, \mathcal{O}_0(\underline{x}_0, \underline{x}_1) \, = \, c_0 \, \alpha_s^3 \, \ln^3 \frac{x_0}{x_1}. \tag{8}$$

Eqs. (7) and (8) describe the perturbative QCD odderon in the color dipole model. Eq. (7) is identical to the dipole model equivalent of the BFKL equation found in [19].

Eq. (7) is illustrated in Fig. 3. The blob on the left hand side of Fig. 3 represents the dipole evolution and the interaction with the target leading to the odderon exchange amplitude for the dipole 01. After one step of the evolution a soft gluon is emitted. This is shown in the three terms on the right hand side of Fig. 3. They all correspond to the three terms on the right of Eq. (7). The first two terms correspond to real emissions, where the subsequent evolution continues either in dipole 02 or in dipole 21. The last term represents virtual corrections, where the gluon can be emitted and absorbed on either side of the blob.

The rapidity variable does not change under C-parity transformation of Eq. (1). Therefore, in the leading logarithmic approximation considered here, charge conjugation only interchanges the transverse coordinates of the quark and the anti-quark. Thus one can explicitly see in Eq. (7) that substituting a C-odd amplitude into its right hand side yields a C-odd expression, as we have seen in the previous section for the case of three-gluon exchange amplitude. Evolution of Eq. (7) preserves C-parity of the amplitude. When the initial condition for Eq. (7) is given by some C-even amplitude, such as the two-gluon exchange, it projects out the C-even solution of this equation, which corresponds to the BFKL pomeron [19]. C-odd initial condition projects out a C-odd solution, which corresponds to the perturbative odderon exchange.

A comment is in order here. In [23] a dipole model equation was constructed which was equivalent to the BKP equation [4] for a four-reggeon system. The resulting equation (48) of [23] is somewhat more complicated than the dipole BFKL equation (7). One may wonder

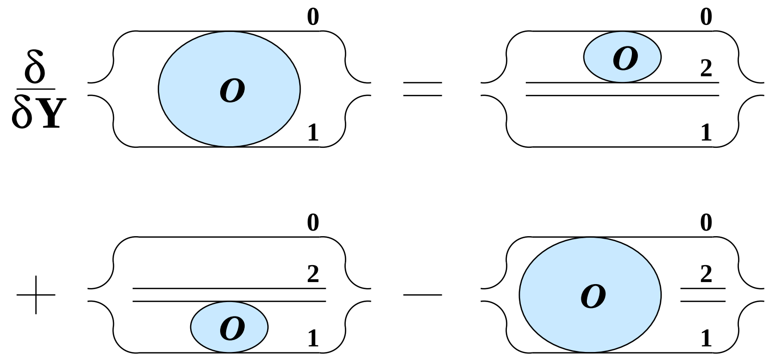

Figure 3: Dipole evolution equation for the odderon exchange amplitude.

why there appears to be no such increase of complication in the dipole model in going from a two-reggeon (pomeron) system to the three reggeon (odderon) system. The reason for that is clear. To describe the four-reggeon state in the dipole model the authors of [23] had to make sure that this state did not mix with the single pomeron exchange, double pomeron exchange and the odderon. Imposing a four-gluon exchange initial conditions in [23] made sure that the state had a positive parity $C = +1$ and did not mix with the odderon having $C = -1$. To avoid mixing with the two-pomeron state the authors of [23] had to make sure that the state had a single-cylinder color topology. Finally, to avoid mixing of this single-cylinder state with the single pomeron the authors of [23] constructed a coupling of four gluons to the dipole wave function which preserved the single-cylinder topology without connecting all four gluons to the same dipole. That required introduction of color quadrupoles and resulted in a more complicated kernel for the integral equation. Now the three-reggeon state considered here is much simpler: the only potential problem is for it to mix with the pomeron, but that can never happen due to different C-parities. Therefore we do not have to invent any complicated color-quadrupole coupling of the three-gluon state to the onium wave function. While it exists, it is N_c-suppressed compared to the dipole coupling considered above.

2.3 Odderon Solution

The solution of Eq. (7) is easy to construct. We first note that the eigenfunctions of the Casimir operators of conformal algebra

$$E^{n,\nu}(\rho_0, \rho_1) \; = \; \left(\frac{\rho_{01}}{\rho_0 \, \rho_1} \right)^{\frac{1+n}{2}+i\nu} \; \left(\frac{\rho_{01}^*}{\rho_0^* \, \rho_1^*} \right)^{\frac{1-n}{2}+i\nu} \tag{9}$$

are also the eigenfunctions of the dipole kernel of Eq. (3) with the eigenvalues

$$2 \, \overline{\alpha}_s \, \chi(n, \nu), \tag{10}$$

where

$$\chi(n, \nu) \; = \; \psi(1) - \frac{1}{2} \, \psi\left(\frac{1 + |n|}{2} + i\nu \right) - \frac{1}{2} \, \psi\left(\frac{1 + |n|}{2} - i\nu \right), \tag{11}$$

$$\overline{\alpha}_s \, = \, \frac{\alpha_s \, N_c}{\pi}, \tag{12}$$

$\rho_{ij} = \rho_i - \rho_j$ and the asterisk in Eq. (9) denotes complex conjugation. Eq. (10) follows from Eq. (A1) which is derived in Appendix A.

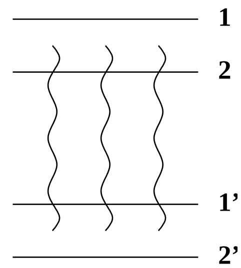

Figure 4: Onium-onium scattering.

To construct the solution of Eq. (7) with a three-gluon exchange initial conditions let us consider onium-onium scattering, i.e. take the target of the above discussion to be a $q\bar{q}$ pair as depicted in Fig. 4. We will work in the frame where the target onium $1'2'$ is at rest, so that all the small-x evolution will be included in the wave function of the incoming onium 12.

A general solution of Eq. (7) is easy to write down once we know that functions $E^{n,\nu}$ from Eq. (9) are the eigenfunctions of its kernel. Since

$$E^{n,\nu}(\rho_1, \rho_2) \, = \, (-1)^n \, E^{n,\nu}(\rho_2, \rho_1) \tag{13}$$

only the functions $E^{n,\nu}(\rho_1, \rho_2)$ with odd n satisfy the condition (2). The most general C-odd solution reads in the complex notation

$$\mathcal{O}(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y) \, = \, \sum_{\text{odd n}} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0 \, e^{2\,\overline{\alpha}_s\,\chi(n,\nu)\,Y} \, C_{n,\nu} \, E^{n,\nu}(\rho_{10}, \rho_{20}) \, E^{n,\nu\,*}(\rho_{1'0}, \rho_{2'0}), \tag{14}$$

where the coefficients $C_{n,\nu}$ have to be determined from the initial conditions. The equivalent of Eq. (8) for the dipole-dipole scattering of Fig. 4 reads in complex notation

$$\mathcal{O}(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y = 0) \, = \, c_0 \, \alpha_s^3 \, \ln^3 \left| \frac{\rho_{11'} \, \rho_{22'}}{\rho_{12'} \, \rho_{1'2}} \right|. \tag{15}$$

To determine the coefficients $C_{n,\nu}$ we rewrite the amplitude (15) as (see Appendix B)

$$\mathcal{O}(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y = 0) \, = \, c_0 \, \alpha_s^3 \, \frac{6}{\pi^2} \sum_{\text{odd n}} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0 \, \frac{\nu^2 + \frac{n^2}{4}}{\left[\nu^2 + \frac{(n+1)^2}{4}\right] \, \left[\nu^2 + \frac{(n-1)^2}{4}\right]}$$
$$\times \, \chi(n, \nu) \, E^{n,\nu}(\rho_{10}, \rho_{20}) \, E^{n,\nu\,*}(\rho_{1'0}, \rho_{2'0}) \tag{16}$$

which yields

$$C_{n,\nu} \, = \, c_0 \, \alpha_s^3 \, \frac{6}{\pi^2} \, \frac{\nu^2 + \frac{n^2}{4}}{\left[\nu^2 + \frac{(n+1)^2}{4}\right] \, \left[\nu^2 + \frac{(n-1)^2}{4}\right]} \, \chi(n, \nu). \tag{17}$$

Using Eq. (17) in Eq. (14) we find the solution of Eq. (7) with the initial condition (15)

$$\mathcal{O}(\rho_1, \rho_2; \rho_{1'}, \rho_{2'}; Y) \, = \, c_0 \, \alpha_s^3 \, \frac{6}{\pi^2} \sum_{\text{odd n}} \int_{-\infty}^{\infty} d\nu \int d^2 \rho_0 \, \frac{\nu^2 + \frac{n^2}{4}}{\left[\nu^2 + \frac{(n+1)^2}{4}\right] \, \left[\nu^2 + \frac{(n-1)^2}{4}\right]}$$

$$\times \, e^{2\,\overline{\alpha}_s \, \chi(n,\nu) \, Y} \, \chi(n,\nu) \, E^{n,\nu}(\rho_{10}, \rho_{20}) \, E^{n,\nu \, *}(\rho_{1'0}, \rho_{2'0}). \tag{18}$$

Therefore, we conclude that the eigenfunctions and eigenvalues of the dipole amplitude with the odderon exchange are given by Eqs. (9) and (10) correspondingly, with only odd values of n. Analyzing Eq. (18) one can easily see that the leading high energy intercept of the odderon amplitude is given by

$$\alpha_{odd} - 1 \, = \, 2 \, \overline{\alpha}_s \, \chi(n=1, \nu=0) \, = \, 0 \tag{19}$$

in agreement with the results of Bartels, Lipatov and Vacca (BLV) [11].

3 Connection to Traditional Approaches

To make a connection between the dipole evolution equation (7) and the BKP equation [4] for the three-reggeon state we will now show that the eigenfunctions from Eq. (9) with odd n correspond to the odderon solution found by Bartels, Lipatov and Vacca in [11]. To find the dipole scattering amplitude generated by the odderon Green function we have to connect the external gluon legs of the Green function to the dipole, as shown in Fig. 5. Instead of the full odderon Green function (see Eq. (5) in [15]), we take a single BLV eigenfunction in the form given by Eq. (2) of [15]

$$\Psi^{n,\nu}(k_1, k_2, k_3) \, = \, c(n,\nu) \sum_{(123)} \frac{(\underline{k}_1 + \underline{k}_2)^2}{\underline{k}_1^2 \, \underline{k}_2^2} \, E^{n,\nu}(\underline{k}_1 + \underline{k}_2, \underline{k}_3), \qquad \text{odd } n, \tag{20}$$

and connect in to the dipole x_{01} in all possible ways as depicted in Fig. 5. $c(n,\nu)$ in Eq. (20) is the overall normalization coefficient which is not important and is given in [15]. Summation over all permutations of 1, 2 and 3 is implied in Eq. (20). The dipole amplitude would then be given by

$$\mathcal{O}^{n,\nu}(\underline{x}_0, \underline{x}_1, Y) \, = \, \int \left[\prod_{i=1}^{3} d^2 k_i \, (e^{-i \underline{k}_i \cdot \underline{x}_0} - e^{-i \underline{k}_i \cdot \underline{x}_1}) \right] \, \Psi^{n,\nu}(k_1, k_2, k_3)$$

$$= \, 3 \, c(n,\nu) \int \left[\prod_{i=1}^{3} d^2 k_i \, (e^{-i \underline{k}_i \cdot \underline{x}_0} - e^{-i \underline{k}_i \cdot \underline{x}_1}) \right] \, \frac{(\underline{k}_1 + \underline{k}_2)^2}{\underline{k}_1^2 \, \underline{k}_2^2} \, E^{n,\nu}(\underline{k}_1 + \underline{k}_2, \underline{k}_3). \tag{21}$$

Substituting

$$E^{n,\nu}(\underline{k}_1 + \underline{k}_2, \underline{k}_3) \, = \, \int \frac{d^2 r_1 \, d^2 r_2}{(2\pi)^4} \, e^{i (\underline{k}_1 + \underline{k}_2) \cdot \underline{r}_1 + i \underline{k}_3 \cdot \underline{r}_2} \, E^{n,\nu}(r_1, r_2) \tag{22}$$

into Eq. (21) and integrating over k_i's yields

$$\mathcal{O}^{n,\nu}(\underline{x}_0, \underline{x}_1, Y) \, = \, 6 \, c(n,\nu) \int d^2 r_1 \, d^2 r_2 \, E^{n,\nu}(r_1, r_2) \, \left\{ \frac{x_{01}^2}{|\underline{x}_1 - \underline{r}_1|^2 \, |\underline{x}_0 - \underline{r}_1|^2} \right.$$

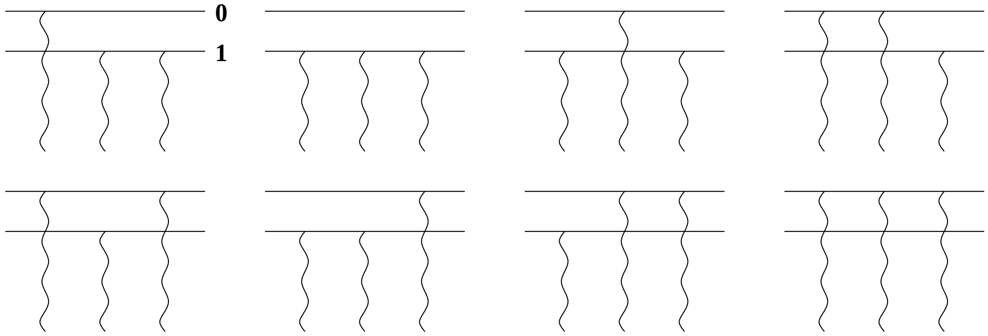

Figure 5: Dipole's impact factor.

$$\times\ [\delta(\underline{x}_1 - \underline{r}_2) - \delta(\underline{x}_0 - \underline{r}_2)] - 2\pi\, \ln \frac{|\underline{x}_1 - \underline{r}_1|}{|\underline{x}_0 - \underline{r}_1|}\, [\delta(\underline{x}_0 - \underline{r}_1)\, \delta(\underline{x}_1 - \underline{r}_2) + \delta(\underline{x}_0 - \underline{r}_2)\, \delta(\underline{x}_1 - \underline{r}_1)] \Bigg\} . \tag{23}$$

Regulating $|\underline{x}_1 - \underline{r}_1| = 0$ and $|\underline{x}_0 - \underline{r}_1| = 0$ singularities in the same way in the first and the second terms on the right hand side of Eq. (23) and integrating Eq. (23) over r_2 we rewrite it as

$$\mathcal{O}^{n,\nu}(\underline{x}_0, \underline{x}_1, Y) \;=\; 6\, c(n,\nu) \int d^2 r_1\, \frac{x_{01}^2}{|\underline{x}_1 - \underline{r}_1|^2\, |\underline{x}_0 - \underline{r}_1|^2}\, [E^{n,\nu}(x_0, r_1) + E^{n,\nu}(r_1, x_1) - E^{n,\nu}(x_0, x_1)] \, . \tag{24}$$

Using Eq. (A1) which is proved in Appendix A we rewrite Eq. (24) as

$$\mathcal{O}^{n,\nu}(\underline{x}_0, \underline{x}_1, Y) \;=\; 24\, c(n,\nu)\, \pi\, \chi(n,\nu)\, E^{n,\nu}(x_0, x_1). \tag{25}$$

Eq. (25) shows that momentum space BLV eigenfunctions in the t-channel Green function translate into the functions $E^{n,\nu}$ for the dipole scattering amplitude in transverse coordinate space up to an overall normalization factor. Therefore our odderon solution (14) is equivalent to the BLV solution [11]. As one can explicitly check, taking the odderon Green function from Eq. (5) of [15], connecting it to the two colliding onia 12 and $1'2'$ as shown at the lowest order in Fig. 4, and Fourier-transforming into transverse coordinate space as was done in arriving at Eq. (25) would yield us the solution in Eq. (18). Starting with the same three-gluon exchange initial condition (16) our evolution equation (7) and the BKP equation [4] would give us the same answer for the amplitude and are equivalent in the case of dipole scattering on a target.

Since the most general C-odd solution of Eq. (7) is known and is given by Eq. (14), we have proven that for the case of a dipole scattering on a target there could be no other odderon solution except for the one found above, or, equivalently, except for BLV solution [11]. Any C-odd dipole amplitude satisfying the condition of Eq. (2) and being a solution of Eq. (7) can always be decomposed in a series (14). Therefore, for the scattering of a dipole on any target the resulting cross section can not grow with center of mass energy s faster than s^0 as follows from the intercept of Eq. (19). (The saddle point evaluation of the ν integral around $\nu = 0$ of the $n = 1$ term in Eq. (18) would give an amplitude which would grow slowly with energy, though the growths would be slower than any power of energy.)

4 Including Saturation Effects

Consider DIS on a proton or a nucleus at high enough energy so that parton saturation effects [21] are becoming important. In [22, 24] (see also [25, 26, 27]) a non-linear evolution equation has been derived which resums all multiple pomeron exchanges in the C-even forward scattering amplitude of a $q\bar{q}$ dipole on the target. The equation is easier to visualize for a nuclear target with atomic number A. As was discussed in [22], in the frame where the nucleus is at rest all the small-x dipole evolution takes place in the incoming $q\bar{q}$ dipole wave function. The evolution, taken in the leading logarithmic approximation, resums powers of $\alpha_s Y$. The color dipoles generated by evolution rescatter on the nuclear target. The rescattering in [22] was given by the Glauber-Mueller formula [28], which corresponds to resummation of the parameter $\alpha_s^2 A^{1/3}$ [29, 30]. This parameter arises if we limit rescattering of a dipole on each nucleon to two-gluon exchange only [30].

To construct a similar equation for the C-odd (odderon) exchange amplitude one has to allow one of the dipoles to interact with one nucleon via a three-gluon exchange. In the high energy regime when $\alpha_s Y \sim 1$ and $\alpha_s^2 A^{1/3} \sim 1$, a single three-gluon exchange would bring in one power of $\alpha_s^3 A^{1/3} \sim \alpha_s$, so that the corresponding odderon exchange amplitude would be of the order $\mathcal{O} \sim \alpha_s \ll 1$. Below we limit our analysis to a single odderon exchange: multiple odderon exchanges would be suppressed by extra powers of α_s. In fact, the C-even amplitude including a pomeron splitting into two odderons which interact with the target would be of the order α_s^2, which is of the same order as the so-called pomeron loop diagrams. Resummation of these diagrams is a very hard unsolved problem (see the discussion in the second reference in [22]) and is beyond the scope of this paper.

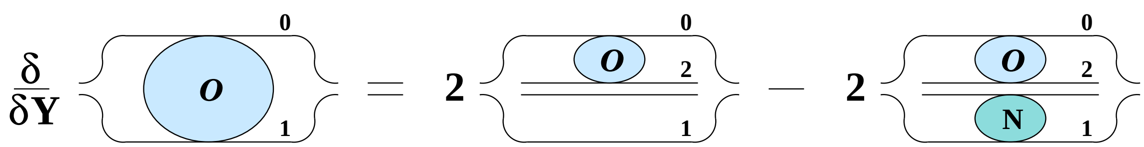

Figure 6: Evolution equation for the odderon exchange amplitude $\mathcal{O}$ in the presence of gluon saturation, which comes in through the C-even dipole ("pomeron") amplitude N.

The evolution equation for the odderon amplitude which includes saturation effects is depicted in Fig. 6. Similar to Fig. 3, in one step of evolution a gluon is emitted splitting the original dipole 01 in two dipoles 02 and 21. The subsequent odderon evolution can take place only in one of the dipoles. The first term on the right hand side of Fig. 6 schematically represents all three terms on the right hand side of Fig. 3 and Eq. (7). The second term on the right hand side of Fig. 6 corresponds to the case where the dipole without the odderon evolution develops nonlinear C-even evolution in it. $N(\underline{x}_0, \underline{x}_1, Y)$ is the C-even forward scattering amplitude of the dipole 01 with rapidity Y on the target including all multiple rescatterings and pomeron exchanges [22]. $N(\underline{x}_0, \underline{x}_1, Y)$ is proportional to the T-matrix of the interaction and is normalized in such a way that it goes to 0 when interactions are weak and goes to 1 in the black limit [22]. The high energy evolution of $N(\underline{x}_0, \underline{x}_1, Y)$ is described by the non-linear evolution equation [22, 24]. Here we assume that N has been found from that evolution equation and is known to us.

The graphical equation from Fig. 6 translates into

$$\frac{\partial}{\partial Y}\mathcal{O}(\underline{x}_0,\underline{x}_1,Y) \;=\; \frac{\alpha_s\,N_c}{2\,\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\,\left[\mathcal{O}(\underline{x}_0,\underline{x}_2,Y)+\mathcal{O}(\underline{x}_2,\underline{x}_1,Y)-\mathcal{O}(\underline{x}_0,\underline{x}_1,Y)\right]$$

$$-\,\frac{\alpha_s\,N_c}{2\,\pi^2}\int d^2x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\,\left[\mathcal{O}(\underline{x}_0,\underline{x}_2,Y)\,N(\underline{x}_2,\underline{x}_1,Y)+N(\underline{x}_0,\underline{x}_2,Y)\,\mathcal{O}(\underline{x}_2,\underline{x}_1,Y)\right]. \tag{26}$$

The initial condition for the amplitude $\mathcal{O}$ in Eq. (26) is given by

$$\mathcal{O}(\underline{x}_0,\underline{x}_1,Y=0) \;=\; c_0\,\alpha_s^3\,\rho\int_{S_\perp} d^2b\,T(\underline{b})\,\ln^3\left(\frac{\underline{B}-\underline{b}+\frac{1}{2}\,\underline{x}_{01}}{\underline{B}-\underline{b}-\frac{1}{2}\,\underline{x}_{01}}\right)\,e^{-x_{01}^2\,Q_s^2(x_{01},B)/4}, \tag{27}$$

where ρ is the atomic number density in the nucleus, $T(\underline{b})$ is the nuclear profile function ($T(\underline{b}) = 2\sqrt{R^2-b^2}$ for a spherical nucleus of radius R) and $\underline{B} = (\underline{x}_0+\underline{x}_1)/2$. It includes a three gluon exchange interaction with a nucleon (modeled by a single quark (cf. Eq. (8))) averaged over transverse positions of the nucleon in the nucleus. (The b-integral in Eq. (27) is not zero since the integration is limited by transverse nuclear size $S_\perp$ and does not go out to infinity.) The exponential factor in Eq. (27) accounts for multiple C-even (two-gluon exchange) rescatterings of the dipole on other nucleons in the nucleus [28, 30], where

$$Q_s^2(x_{01},B) = 4\pi\,\alpha_s^2\,\frac{C_F}{N_c}\,\rho\,T(\underline{B})\,\ln\frac{1}{x_{01}\,\Lambda} \tag{28}$$

with Λ some infrared cutoff.

Eq. (26) describes the odderon evolution including the effects of saturation in the leading logarithmic approximation. While a detailed analysis of Eq. (26) is beyond the scope of this paper, we note that gluon saturation introduces negative terms on the right hand side of Eq. (26) (the last two terms in it). Since the leading high energy intercept of Eq. (7) corresponding to the first three terms on the right hand side of (26) is zero, the negative non-zero last two terms on the right hand side of (26) are likely to make the odderon amplitude a decreasing function of energy/rapidity. This result may be relevant to the lack of observation of QCD odderon in the high energy DIS and pp data.

5 Discussion

In this paper we have shown that in the framework of the dipole model both the Pomeron and the Odderon are described by the same evolution equation (7). While the Pomeron evolution is projected out of Eq. (7) by C-even initial conditions [19], the Odderon evolution is projected out by the C-odd ones. Since the complete set of solutions to the BFKL equation is known, we have constructed the most general Odderon solution for the dipole amplitude given by Eq. (14). We have thus showed that there could be no other Odderon solution for the case of a dipole scattering on any target. Our solution (14) is equivalent to the BLV solution [11], as shown in Sect. 3.

In the usual Feynman diagram language the dipole model picks out a part of the Odderon Green function with at least two out of three t-channel gluons connecting to the same (anti)-quark line. Therefore at least two gluons have the same transverse coordinate. This happens for

the Odderon exchange in DIS. However, in the case of pp or $p\bar{p}$ scattering there exist Feynman diagrams with the three t-channel gluons coupling to different quarks at different transverse coordinates. These diagrams have no equivalent in the dipole model. One can not reproduce the solution of [9] in our approach since it vanishes when two gluons are taken at the same transverse point. While the dipole model allowed us to find the most general Odderon solution for the case of dipole scattering on any target, it did not produce any upper bound on the intercept of the Odderon exchanged in the scattering of two three-quark states (pp or $p\bar{p}$).

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Appendix A: Eigenfunctions of the Dipole Kernel

In this appendix we show that functions $E^{n,\nu}(x_0, x_1)$ are eigenfunctions of the dipole kernel, such that

$$\int d^2x_2 \, \frac{x_{01}^2}{x_{02}^2 \, x_{12}^2} \, [E^{n,\nu}(x_0, x_2) + E^{n,\nu}(x_2, x_1) - E^{n,\nu}(x_0, x_1)] \; = \; 4\,\pi\,\chi(n,\nu)\, E^{n,\nu}(x_0, x_1) \qquad \text{(A1)}$$

with $\chi(n,\nu)$ given by Eq. (11). Our derivation below will be closely following the Appendix of [5] (see also [31]). In the complex notation the integral (A1) can be written as

$$I(\rho_0, \rho_0^*, \rho_1, \rho_1^*) \; \equiv \; \int d^2\rho_2 \, \frac{|\rho_{01}|^2}{|\rho_{02}|^2 \, |\rho_{12}|^2} \, [E^{n,\nu}(\rho_0, \rho_2) + E^{n,\nu}(\rho_2, \rho_1) - E^{n,\nu}(\rho_0, \rho_1)] \qquad \text{(A2)}$$

where the asterisk denotes complex conjugation. Performing the inversion transformation $\rho_i \to 1/\rho_i$ yields

$$I(1/\rho_0, 1/\rho_0^*, 1/\rho_1, 1/\rho_1^*) \; = \; \int d^2\rho_2 \, \frac{|\rho_{01}|^2}{|\rho_{02}|^2 \, |\rho_{12}|^2} \, \left[\rho_{20}^h \, \rho_{20}^{*\,\bar{h}} + \rho_{12}^h \, \rho_{12}^{*\,\bar{h}} - \rho_{10}^h \, \rho_{10}^{*\,\bar{h}}\right] \qquad \text{(A3)}$$

where $h = \frac{1+n}{2} + i\nu$ and $\bar{h} = \frac{1-n}{2} + i\nu$. Defining a new variable

$$R \; = \; \frac{1}{2}\,(\rho_{20} + \rho_{21}) \qquad \text{(A4)}$$

we rewrite Eq. (A3) as

$$I(1/\rho_0, 1/\rho_0^*, 1/\rho_1, 1/\rho_1^*) \; = \; \int d^2R \, \frac{|\rho_{01}|^2}{|R - \frac{1}{2}\rho_{01}|^2 \, |R + \frac{1}{2}\rho_{01}|^2} \, \Big[|R - \frac{1}{2}\rho_{01}|^{1+2i\nu}$$

$$\times \left(\frac{R - \frac{1}{2}\rho_{01}}{R^* - \frac{1}{2}\rho_{01}^*}\right)^{n/2} + (-1)^n \, |R + \frac{1}{2}\rho_{01}|^{1+2i\nu} \left(\frac{R + \frac{1}{2}\rho_{01}}{R^* + \frac{1}{2}\rho_{01}^*}\right)^{n/2} - |\rho_{10}|^{1+2i\nu} \left(\frac{\rho_{10}}{\rho_{10}^*}\right)^{n/2} \Bigg]. \quad \text{(A5)}$$

Redefining the integration variable again such that $R = z\rho_{01}/2$ we obtain

$$I(1/\rho_0, 1/\rho_0^*, 1/\rho_1, 1/\rho_1^*) \; = \; 4\,|\rho_{10}|^{1+2i\nu} \left(\frac{\rho_{10}}{\rho_{10}^*}\right)^{n/2} \int d^2 z \, \frac{1}{(z^2-1)\,(z^{*\,2}-1)} \Bigg[(-1)^n \, 2^{-1-2i\nu}$$

$$\times \; |z-1|^{1+2i\nu} \left(\frac{z-1}{z^*-1}\right)^{n/2} + 2^{-1-2i\nu} \, |z+1|^{1+2i\nu} \left(\frac{z+1}{z^*+1}\right)^{n/2} - 1\Bigg]. \quad \text{(A6)}$$

Undoing the inversion transformation $\rho_i \to 1/\rho_i$ in Eq. (A6) gives

$$I(\rho_0, \rho_0^*, \rho_1, \rho_1^*) \; = \; C(n,\nu)\, E^{n,\nu}(\rho_0, \rho_1) \quad \text{(A7)}$$

with

$$C(n,\nu) \; = \; 4 \int d^2 z \, \frac{1}{(z^2-1)\,(z^{*\,2}-1)} \Bigg[(-1)^n \, 2^{-1-2i\nu} \, |z-1|^{1+2i\nu} \left(\frac{z-1}{z^*-1}\right)^{n/2}$$

$$+ \; 2^{-1-2i\nu} \, |z+1|^{1+2i\nu} \left(\frac{z+1}{z^*+1}\right)^{n/2} - 1\Bigg]. \quad \text{(A8)}$$

Eq. (A7) proves that functions $E^{n,\nu}(\rho_0, \rho_1)$ are, indeed, eigenfunctions of the dipole kernel. To find the eigenvalue $C(n,\nu)$ we follow the steps outlined in the Appendix of [5]. Rewriting $z = x + iy$ with real x and y we perform Wick rotation in the complex y-plane. The y-integral transforms into an integral over $t = -iy$. Following [5] we define $\alpha = x - t$ and $\beta = x + t$ obtaining

$$C(n,\nu) \; = \; 2\,i \lim_{\sigma\to 0} \int_{-\infty}^{\infty} d\alpha \, d\beta \, \frac{1}{[(1-\alpha)(1-\beta) + i\epsilon]^{1-\sigma} \, [(1+\alpha)(1+\beta) + i\epsilon]^{1-\sigma}}$$

$$\times \; \Big\{ 2^{-1-2i\nu} \, (1-\alpha)^{|n|} \, [(1-\alpha)(1-\beta) + i\epsilon]^{\frac{1-|n|}{2} + i\,\nu}$$

$$+ 2^{-1-2i\nu} \, (1+\alpha)^{|n|} \, [(1+\alpha)(1+\beta) + i\epsilon]^{\frac{1-|n|}{2} + i\,\nu} - 1 \Big\}, \quad \text{(A9)}$$

where we introduced a dimensional regulator σ to be able to integrate the terms separately. The β-contour in Eq. (A9) can be distorted to give 0 unless $-1 < \alpha < 1$. Noting that the first and the second terms in the curly brackets of Eq. (A9) are identical we rewrite it as

$$C(n,\nu) \; = \; 2\,i \lim_{\sigma\to 0} \int_{-1}^{1} d\alpha \, \frac{1}{(1-\alpha^2)^{1-\sigma}} \int_{-\infty}^{\infty} d\beta \, \frac{1}{(1-\beta+i\epsilon)^{1-\sigma}\,(1+\beta+i\epsilon)^{1-\sigma}}$$

$$\times \; \Big\{ 2^{-2i\nu} \, (1-\alpha)^{\frac{1+|n|}{2} + i\,\nu} \, (1-\beta+i\epsilon)^{\frac{1-|n|}{2} + i\,\nu} - 1 \Big\}. \quad \text{(A10)}$$

The β-integral in Eq. (A10) can be done by distorting the contour around one of the branch cuts. Performing α-integral as well and taking $\sigma \to 0$ limit of Eq. (A10) yields

$$C(n,\nu) \; = \; 4\,\pi\,\chi(n,\nu). \quad \text{(A11)}$$

Combining Eqs. (A7) and (A11) with Eq. (A2) gives us Eq. (A1), as desired.

Appendix B: Three-Gluon Exchange Amplitude

Here we decompose the function

$$\ln^3 \left| \frac{\rho_{11'}\, \rho_{22'}}{\rho_{12'}\, \rho_{1'2}} \right| \tag{B1}$$

in the series of Eq. (16). First we note that

$$\nabla_1^2\, \nabla_2^2\, \ln^3 \left| \frac{\rho_{11'}\, \rho_{22'}}{\rho_{12'}\, \rho_{1'2}} \right| = 6\,\pi\, [\delta(\rho_{22'}) - \delta(\rho_{21'})] \int d^2\rho_3\, \frac{|\rho_{1'2'}|^2}{|\rho_{31'}|^2\, |\rho_{32'}|^2}\, [2\,\delta(\rho_{31}) - \delta(\rho_{11'})]$$

$$+ 6\,\pi\, [\delta(\rho_{11'}) - \delta(\rho_{12'})] \int d^2\rho_3\, \frac{|\rho_{1'2'}|^2}{|\rho_{31'}|^2\, |\rho_{32'}|^2}\, [2\,\delta(\rho_{32}) - \delta(\rho_{22'})]\,. \tag{B2}$$

Then, employing (see Eq. (25) in [5])

$$(2\pi)^4\, \delta(\rho_{11'})\, \delta(\rho_{22'}) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0\, \frac{16\left(\nu^2 + \frac{n^2}{4}\right)}{|\rho_{12}|^2\, |\rho_{1'2'}|^2}\, E^{n,\nu}(\rho_{10}, \rho_{20})\, E^{n,\nu\,*}(\rho_{1'0}, \rho_{2'0}) \tag{B3}$$

in Eq. (B2) and remembering that (see Eqs. (21) of [5])

$$\nabla_1^2\, \nabla_2^2\, E^{n,\nu}(\rho_{10}, \rho_{20}) = \frac{16}{|\rho_{12}|^4} \left[\nu^2 + \frac{(n+1)^2}{4}\right] \left[\nu^2 + \frac{(n-1)^2}{4}\right] E^{n,\nu}(\rho_{10}, \rho_{20}) \tag{B4}$$

yields

$$\ln^3 \left| \frac{\rho_{11'}\, \rho_{22'}}{\rho_{12'}\, \rho_{1'2}} \right| = \frac{3}{2\,\pi^3} \sum_{\text{odd n}} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0\, \frac{\nu^2 + \frac{n^2}{4}}{\left[\nu^2 + \frac{(n+1)^2}{4}\right] \left[\nu^2 + \frac{(n-1)^2}{4}\right]}\, E^{n,\nu}(\rho_{10}, \rho_{20})$$

$$\times \int d^2\rho_3\, \frac{|\rho_{1'2'}|^2}{|\rho_{31'}|^2\, |\rho_{32'}|^2}\, [E^{n,\nu\,*}(\rho_{1'0}, \rho_{30}) + E^{n,\nu\,*}(\rho_{30}, \rho_{2'0}) - E^{n,\nu\,*}(\rho_{1'0}, \rho_{2'0})] \tag{B5}$$

Using Eq. (A1) to evaluate Eq. (B5) gives the final answer

$$\ln^3 \left| \frac{\rho_{11'}\, \rho_{22'}}{\rho_{12'}\, \rho_{1'2}} \right| = \frac{6}{\pi^2} \sum_{\text{odd n}} \int_{-\infty}^{\infty} d\nu \int d^2\rho_0\, \frac{\nu^2 + \frac{n^2}{4}}{\left[\nu^2 + \frac{(n+1)^2}{4}\right] \left[\nu^2 + \frac{(n-1)^2}{4}\right]}$$

$$\times\, \chi(n,\nu)\, E^{n,\nu}(\rho_{10}, \rho_{20})\, E^{n,\nu\,*}(\rho_{1'0}, \rho_{2'0}). \tag{B6}$$

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